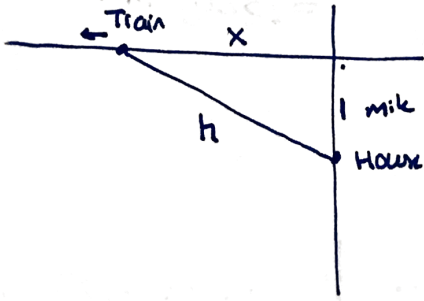


①



Relation:

$$x^2 + 1 = h^2$$

Differentiated:

$$2x \cdot x' = 2h \cdot h'$$

Given info. (at key moment):

$$x' = 40 \text{ (mph)}$$

$$x = 40 \text{ mph} \cdot 6 \text{ min} \\ = 40 \frac{\text{m}}{\text{hr}} \cdot \frac{1}{10} \text{ hr} = 4 \text{ m}$$

$$\Rightarrow h = \sqrt{1+x^2} = \sqrt{1+4^2} = \sqrt{17}$$

substituting $x'=40$, $x=4$, $h=\sqrt{17}$ gives

$$2 \cdot 4 \cdot 40 = 2\sqrt{17} \cdot h'$$

$$\Rightarrow h' = \frac{2 \cdot 4 \cdot 40}{2 \cdot \sqrt{17}} = \boxed{\frac{160}{\sqrt{17}}} \text{ (mph)} \quad (\approx 38.8 \text{ mph})$$

②



$$V = \frac{4}{3} \pi r^3$$

$$V' = \frac{4}{3} \cdot \pi \cdot 3r^2 \cdot r'$$

$$V' = 4\pi r^2 \cdot r'$$

At this moment:

$$r = 2$$

$$\Rightarrow V = \frac{32}{3} \pi$$

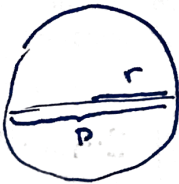
$$r' = -1$$

sub.:

$$V' = 4\pi \cdot 2^2 \cdot (-1)$$

$$= \boxed{-16\pi} \text{ in}^3/\text{min}$$

③



$$A = \pi r^2$$

$$A' = 2\pi r \cdot r'$$

Given:

$$r = 10 \text{ (diam 20)}$$

$$A' = 100 \text{ ft}^2/\text{min}$$

sub.:

$$100 = 2\pi \cdot 10 \cdot r'$$

$$r' = \frac{100}{20\pi} = \boxed{\frac{5}{\pi}} \text{ ft/min.}$$

④



$$A = w \cdot h$$

$$A' = w'h + wh'$$

Given

$$w' = 2 \quad w = 6$$

$$h' = -3 \quad h = 8$$

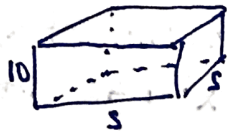
sub.

$$A' = 2 \cdot 8 - 6 \cdot 3$$

$$= 16 - 18$$

$$= \boxed{-2} \text{ in}^2/\text{sec.}$$

5



$$V = 10s^2$$

$$V' = 20s \cdot s'$$

Given
 $s = 8$
 $s' = 2$

$$V' = 20 \cdot 8 \cdot 2$$

$$= \boxed{320} \text{ in}^3/\text{min.}$$

6



$$V = \frac{1}{3} \pi r^2 h$$

when $\frac{h}{r} = \frac{12}{8} = 2$
 i.e. $h = \frac{12}{7} r$

$$r' = -2$$

$$r = 2$$

$$V' = \frac{12}{7} \pi \cdot 2^2 \cdot (-2)$$

$$= \frac{12}{7} \pi \cdot (-8)$$

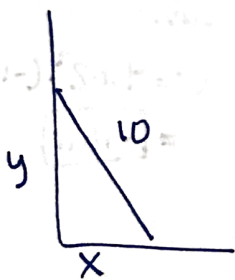
$$V = \frac{1}{3} \pi r^2 \cdot \frac{12}{7} r$$

$$= \boxed{-\frac{96}{7} \pi} \text{ ft}^3/\text{min}$$

$$V = \frac{4}{7} \pi r^3$$

$$V' = \frac{12}{7} \pi r^2 \cdot r'$$

7



$$x^2 + y^2 = 10^2$$

$$2x \cdot x' + 2y \cdot y' = 0$$

$$y' = -1$$

$$y = 3$$

$$\Rightarrow \cancel{x} = 4$$

$$x = \sqrt{91}$$

$$2 \cdot \sqrt{91} \cdot x' + 2 \cdot 3 \cdot (-1) = 0$$

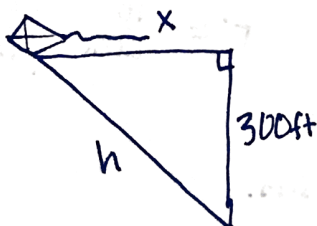
$$2\sqrt{91} x' = 6$$

$$x' = \frac{3}{\sqrt{91}} \text{ ft/sec.}$$

key moment:



8



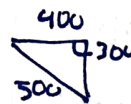
$$x^2 + 300^2 = h^2$$

$$2x \cdot x' = 2h \cdot h'$$

$$x' = 10$$

$$x = 10 \frac{\text{ft}}{\text{sec}} \cdot 40 \text{ sec} = 400$$

$$\Rightarrow h = 500$$



$$2 \cdot 400 \cdot 10 = 2 \cdot 500 \cdot h'$$

$$h' = \frac{400}{500} \cdot 10$$

$$= \boxed{8} \text{ ft/sec.}$$