WORKSHEET 2 SOLUTIONS:

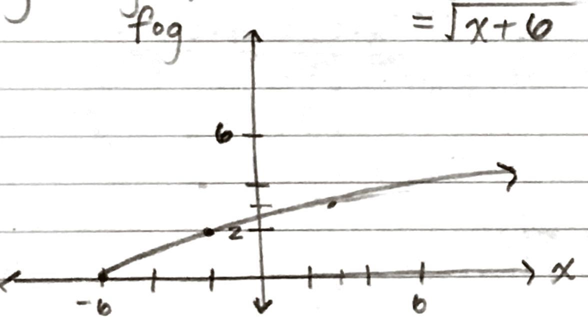
1. $g(x) = \dfrac{\dfrac{x}{x-1} - \dfrac{x+2}{x}}{x-2} \Rightarrow \dfrac{\dfrac{x(x)}{x(x-1)} - \dfrac{(x-1)(x+2)}{x(x-1)}}{x-2} \Rightarrow \dfrac{\dfrac{x^2}{x(x-1)} - \dfrac{x^2+x-2}{x(x-1)}}{x-2}$

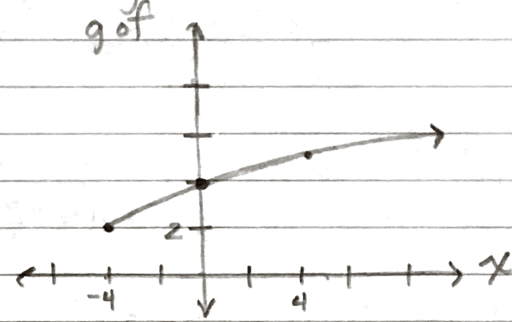
$= \dfrac{\dfrac{x^2-x^2-x+2}{x(x-1)}}{x-2} = \dfrac{\dfrac{-x+2}{x(x-1)}}{x-2} = \dfrac{-x+2}{x(x-1)} \cdot \dfrac{1}{x-2} = \dfrac{-\cancel{(x-2)}}{x(x+1)} \cdot \dfrac{1}{\cancel{(x-2)}} = \boxed{\dfrac{-1}{x^2-x}}$

2. $f \circ g(x) = f(g(x))$

(a). Note that the domain of a function is everything the function can take in. So, the domain of $f(g(x))$ will be defined by $g(x)$ – that's what goes in. Since $g(x)$ is a value, specifically any value given by plugging in xs, the domain of $f(g(x))$ is the range of $g(x)$.

(b) $f \circ g = f(g(x)) = f(x+2) = \sqrt{(x+2)+4} = \sqrt{x+6}$ | $g \circ f = g(f(x)) = g(\sqrt{x+4}) = \sqrt{x+4} + 2$





$f \circ g$ does NOT equal $g \circ f$. Consider $x=0$ $\left.\begin{array}{l} f(g(0)) = \sqrt{6} \\ g(f(0)) = 4 \end{array}\right\}$ These are not equal!

3. $f(f(2)) = \dfrac{f(2)+1}{f(2)-1}$ → You can break this up! $f(2) = \dfrac{2+1}{2-1} = \dfrac{3}{1} = 3$

* Remember to plug all of this in first!

→ So $\dfrac{f(2)+1}{f(2)-1} = \dfrac{3+1}{3-1} = \dfrac{4}{2} = 2$

$f(f(x)) = \dfrac{f(x)+1}{f(x)-1} = \dfrac{\dfrac{x+1}{x-1}+1}{\dfrac{x+1}{x-1}-1} = \dfrac{\dfrac{x+1}{x-1} + \dfrac{x-1}{x-1}}{\dfrac{x+1}{x-1} - \left(\dfrac{x-1}{x-1}\right)} = \dfrac{\dfrac{x+1+x-1}{x-1}}{\dfrac{x+1-x+1}{x-1}} = \dfrac{\dfrac{2x}{x-1}}{\dfrac{2}{x-1}}$

$= \dfrac{2x}{\cancel{x-1}} \cdot \dfrac{\cancel{x-1}}{2} = \dfrac{2x}{2} = \boxed{x}$

4. $f(x)=\dfrac{1}{x+1} \Rightarrow \dfrac{f(x+h)-f(x)}{h} = \dfrac{\dfrac{1}{(x+h)+1} - \dfrac{1}{x+1}}{h} = \dfrac{\dfrac{x+1}{(x+1)(x+h+1)} - \dfrac{x+h+1}{(x+1)(x+h+1)}}{h}$

$= \dfrac{\dfrac{\cancel{x}+\cancel{1}-\cancel{x}-h-\cancel{1}}{(x+1)(x+h+1)}}{h} = \dfrac{\dfrac{-h}{(x+1)(x+h+1)}}{h} = \dfrac{-\cancel{h}}{(x+1)(x+h+1)} \cdot \dfrac{1}{\cancel{h}} = \boxed{\dfrac{-1}{(x+1)(x+h+1)}}$

5. $f(x)=\dfrac{x-7}{x+3} \Rightarrow \dfrac{f(x+h)-f(x)}{h} = \dfrac{\dfrac{x+h-7}{x+h+3} - \dfrac{x-7}{x+3}}{h} = \dfrac{\dfrac{(x+3)(x+h-7)}{(x+3)(x+h+3)} - \dfrac{(x-7)(x+h+3)}{(x+3)(x+h+3)}}{h}$

* Remember to distribute

$= \dfrac{\dfrac{\cancel{x^2}+\cancel{xh}-\cancel{7x}+\cancel{3x}+3h-\cancel{21}}{(x+3)(x+h+3)} - \dfrac{\cancel{x^2}+\cancel{xh}+\cancel{3x}-\cancel{7x}-7h-\cancel{21}}{(x+3)(x+h+3)}}{h}$

$= \dfrac{\dfrac{3h+7h}{(x+3)(x+h+3)}}{h} = \dfrac{10\cancel{h}}{(x+3)(x+h+3)} \cdot \dfrac{1}{\cancel{h}} = \boxed{\dfrac{10}{(x+3)(x+h+3)}}$

6.(a) $\dfrac{x^2+6x+8}{x^2-4} = \dfrac{\cancel{(x+2)}(x+4)}{\cancel{(x+2)}(x-2)} = \boxed{\dfrac{x+4}{x-2}}$

(b) $\dfrac{x^2+6x+8}{x^2-5x-14} = \dfrac{\cancel{(x+2)}(x+4)}{(x-7)\cancel{(x+2)}} = \boxed{\dfrac{x+4}{x-7}}$

(c) $\dfrac{x^2-6x+8}{x^2-x-2} = \dfrac{(x-4)\cancel{(x-2)}}{\cancel{(x-2)}(x+1)} = \boxed{\dfrac{x-4}{x+1}}$

(d) $\dfrac{1}{t\sqrt{1+t}} - \dfrac{1}{t} = \dfrac{1}{t\sqrt{1+t}} - \dfrac{\sqrt{1+t}}{t\sqrt{1+t}} = \boxed{\dfrac{1-\sqrt{1+t}}{t\sqrt{1+t}}}$

(e) $\dfrac{t-1}{g(t^2)-3} = \dfrac{t-1}{2t^2+1-3} = \dfrac{t-1}{2t^2-2} = \dfrac{t-1}{2(t^2-1)} = \dfrac{\cancel{t-1}}{2\cancel{(t-1)}(t+1)} = \boxed{\dfrac{1}{2(t+1)}}$

(f) $\dfrac{x^2-13x+42}{x^2-4x+12} = \dfrac{(x-7)(x-6)}{x^2-4x+12}$ } can't simplify anymore?

Check: $\dfrac{4 \pm \sqrt{16-4(12)}}{2}$

$= \dfrac{4 \pm \sqrt{-32}}{2}$ ← this neg!

This tells us that there are no real solutions to this, so it can't be simplified more!

(g) $\frac{1}{x} - \frac{1}{|x|}$ $\Rightarrow$ Case 1: $|x|$, x is pos.
↳ $\frac{1}{x} - \frac{1}{x} = \frac{0}{x} = \boxed{0}$

Case 2: $|x|$, x is neg.
↳ $\frac{1}{x} - \left(\frac{1}{-x}\right) = \frac{1}{x} + \frac{1}{x} = \boxed{\frac{2}{x}}$

h) $\frac{|x+4|}{x+4}$ $\Rightarrow$ Case 1: $|x+4|$, $x+4$ is pos.
↳ $\frac{x+4}{x+4} = \boxed{1}$

Case 2: $|x+4|$, $x+4$ is neg.
↳ $\frac{-(x+4)}{x+4} = \boxed{-1}$

(i) $f(x) = \frac{1}{x}$, $\frac{f(t-1) - 2f(t)}{t^2-4} = \frac{\frac{1}{t-1} - 2\left(\frac{1}{t}\right)}{t^2-4} = \frac{\frac{t}{(t-1)t} - \frac{2(t-1)}{t(t-1)}}{t^2-4} = \frac{\frac{t-2t+2}{t(t-1)}}{t^2-4}$

$= \frac{\frac{-t+2}{t(t-1)}}{t^2-4} = \frac{-\cancel{(t-2)}}{t(t-1)} \cdot \frac{1}{\cancel{(t-2)}(t+2)} = \boxed{\frac{-1}{t(t-1)(t+2)}}$

7. $f(g(x)) = \frac{x^3+1}{x^3+2}$

Note: For problems like this one, try to find a repeating function that would "be plugged in".

↳ $g(x) = x^3$ and $f(x) = \frac{x+1}{x+2}$ $\Rightarrow$ $f(g(x)) = \frac{x^3+1}{x^3+2}$ and $g(f(x)) = \left(\frac{x+1}{x+2}\right)^3$

8 (a) Consider: $\frac{xy+x}{xz} = \frac{\cancel{x}(y+1)}{\cancel{x}z} = \frac{y+1}{z}$ } This is correct b/c the x can be factored out and cancelled

$\frac{xy+y^2}{xz} \neq \frac{y+y^2}{z}$ } This is incorrect b/c we can't factor out an x in $xy+y^2$. So we cancel the x in the denominator.

(b) Consider $\sqrt{x^2y^4} = \sqrt{(xy^2)^2} = xy^2$ } This is correct b/c there are 2 copies of xy^2 in x^2y^4, so we can simplify the radical.

NOTE: $\sqrt{a^{2n}} = \sqrt{(a^n)^2} = a^n$

Consider $\sqrt{x^2+y^4} \neq x+y^2$ } The mistake made here was simplifying each individual term. In order for this to be true, we need $(x+y^2)^2 = x^2+y^4$ (what's under the radical).
↳ $(x+y^2)(x+y^2) = x^2+2xy^2+y^4 \neq x^2+y^4$

NOTE: $\sqrt{a^2+b^2} \neq \sqrt{a^2} + \sqrt{b^2}$