Worksheet 1: Solutions

1 (a) $\dfrac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \left(\frac{a}{b}\right) \times \left(\frac{d}{c}\right) = \frac{ad}{bc}$

(b) $\dfrac{1}{\left(\frac{a}{b}\right)} = 1 \times \left(\frac{b}{a}\right) = \frac{b}{a}$

(c) $\dfrac{\left(\frac{a}{b}\right)}{c} = \left(\frac{a}{b}\right) \times \frac{1}{c} = \frac{a}{bc}$

(d) $\dfrac{a}{\left(\frac{b}{c}\right)} = \frac{a}{1} \times \left(\frac{c}{b}\right) = \frac{ac}{b}$

2. $1 + \dfrac{1}{1+\frac{1}{x}} \Rightarrow 1 + \dfrac{1}{\frac{x}{x} + \frac{1}{x}}$

$= 1 + \dfrac{1}{\frac{x+1}{x}} = 1 + \dfrac{x}{x+1} = \dfrac{x+1}{x+1} + \dfrac{x}{x+1} = \dfrac{x+1+x}{x+1} = \boxed{\dfrac{2x+1}{x+1}}$

3 (a) $x^2 - 4x - 21 = 0 \Rightarrow (x-7)(x+3) = 0 \Rightarrow x = 7$ OR $x = -3$

can't easily factor, so:

(b) $x^2 - x + 7 = 0 \Rightarrow \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{1 \pm \sqrt{(-1)^2 - 4(1)(+7)}}{2(1)} = \dfrac{1 \pm \sqrt{1-28}}{2} = \dfrac{1 \pm \sqrt{-27}}{2}$ can't factor b/c $\sqrt{-27}$

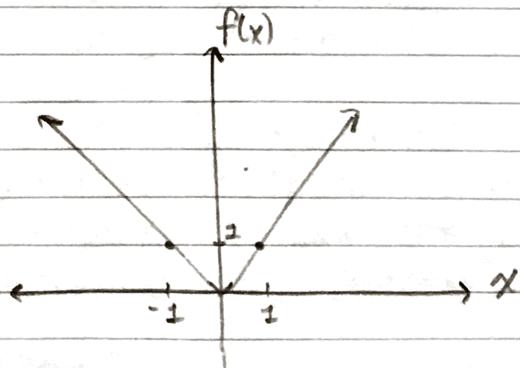
(c) $x^2 + 2x - 4 = 0 \Rightarrow \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(-4)}}{2} = \dfrac{-2 \pm \sqrt{20}}{2} = \dfrac{-2 \pm 2\sqrt{5}}{2} = \boxed{-1 \pm \sqrt{5}}$

(d) $x^3 - 5x^2 + 6x = 0 \Rightarrow x(x^2 - 5x + 6) = 0 \Rightarrow x(x-3)(x-2) = 0 \Rightarrow x = 3$ OR $x = 2$ OR $x = 0$

4 $\sqrt{x^2+4} \neq x + 2$. Take $x = 1$. $\sqrt{1^2+4} = \sqrt{5}$ and $1 + 2 = 3$
But we know that $\sqrt{5} \neq 3$. So it is not always true!

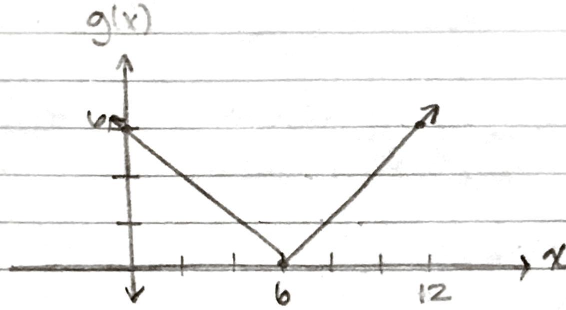
5. $f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

(a) Domain = $\mathbb{R}$ or $(-\infty, \infty)$
Range = $\mathbb{R}^+$ or $(0, \infty)$



Near $x = 0$, this function is getting close to $y = 0$.
$x = 0$ is a corner point

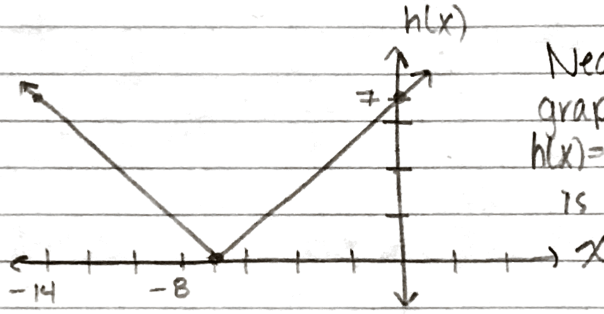
(b) $g(x) = \begin{cases} -(x-6) & \text{if } x < 6 \\ x-6 & \text{if } x \geq 6 \end{cases}$



Near $x=6$, this graph is going to $g(x)=0$. $x=6$ is a CORNER point

This graph relates to the graph $f(x)=|x|$ because it is the same graph shifted to the right by 6.

(c) $h(x) = \begin{cases} -(x+7) & \text{if } x < -7 \\ x+7 & x \geq -7 \end{cases}$



Near $x=-7$, this graph is going to $h(x)=0$. $x=-7$ is a corner pt.

This graph relates to the graph $f(x)=|x|$ because it is the same graph shifted to the left by 7.

6. To find the perpendicular slope of $2x+5y=6$

$\Rightarrow 5y = -2x+6 \Rightarrow y = -2/5 x + 6/5$

So we need $m = 5/2$

Now that we have our slope and point we use point-slope formula:

$y - y_1 = m(x - x_1) \Rightarrow (y+1) = 5/2(x-3)$

$\Rightarrow y+1 = 5/2 x - 15/2 \Rightarrow y = \boxed{5/2 x - 17/2}$

To check if the line passes through $(1,-6)$:

Set $-6 \stackrel{?}{=} 5/2(1) - 17/2 = -12/2 = -6$ ✓. The line does pass through!

7. $f(x) = x^2 - 6x - 7$

(a) $f(0) = 0^2 - 6(0) - 7 = -7$

(b) $f(-3) = (-3)^2 - 6(-3) - 7 = 9 + 18 - 7 = 20$

(c) $f(1) = (1)^2 - 6(1) - 7 = 1 - 6 - 7 = -12$

(d) For what values does $f(x)=0$?

To solve, set $f(x) = x^2 - 6x - 7 = 0$

$\Rightarrow (x-7)(x+1) = 0$, so we have $(x-7)=0$ or $(x+1)=0$

$\boxed{f(x)=0 \text{ for } x=7 \text{ and } x=-1}$

(e) $f(a) = a^2 - 6a - 7$

(f) $f(a+h) = (a+h)^2 - 6(a+h) - 7$

$= (a+h)(a+h) - 6(a+h) - 7 = a^2 + 2ah + h^2 - 6a - 6h - 7$

(g) $\frac{f(a+h) - f(a)}{h} \rightarrow$ Notice that we already computed $f(a+h)$ and $f(a)$.

$= \frac{a^2 + 2ah + h^2 - 6a - 6h - 7 - (a^2 - 6a - 7)}{h}$

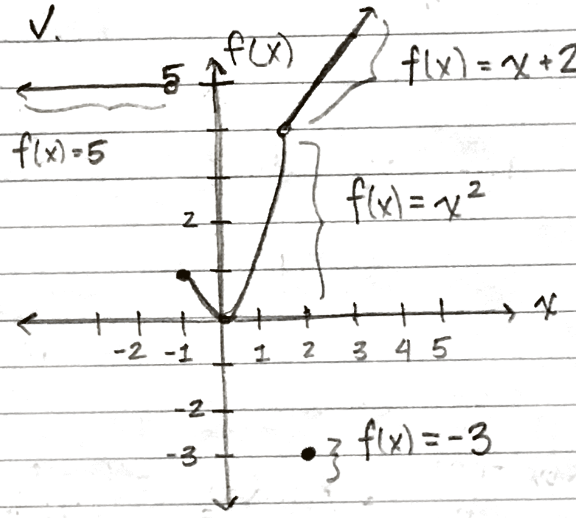
$= \frac{\cancel{a^2} + 2ah + h^2 \cancel{-6a} - 6h \cancel{-7} \cancel{-a^2} \cancel{+6a} \cancel{+7}}{h} = \frac{h^2 + 2ah - 6h}{h} = \frac{\cancel{h}(h+2a-6)}{\cancel{h}}$

this is treated as our input!

(h) First, note that for $f(f(x))$. So once you input (whatever is in here) you can evaluate. Remember though, we know what $f(x)$ is! Our given equation!!

$$f(f(x)) = (f(x))^2 - 6(f(x)) - 7$$
$$= (x^2-6x-7)^2 - 6(x^2-6x-7) - 7 = (x^2-6x-7)(x^2-6x-7) - 6(x^2-6x-7) - 7$$
$$= x^4 - 6x^3 - 7x^2 - 6x^3 + 36x^2 + 42x - 7x^2 + 42x + 49 - 6x^2 + 36x + 42 - 7.$$
$$= x^4 - 6x^3 - 6x^3 - 7x^2 + 36x^2 - 7x^2 - 6x^2 + 42x + 42x + 36x + 49 + 42 - 7$$
$$= x^4 - 12x^3 + 16x^2 + 120x + 84. \quad \checkmark$$

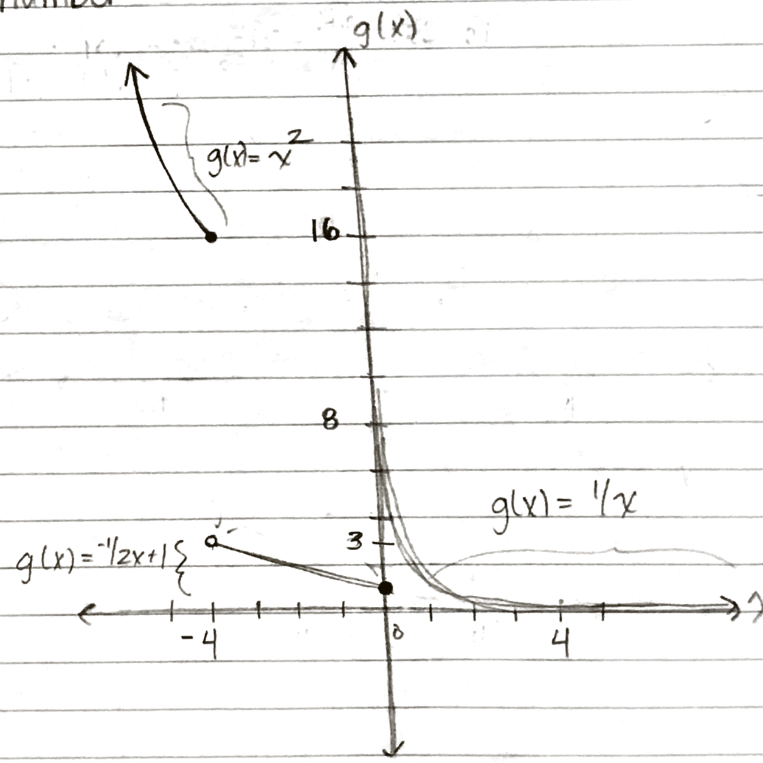
8 $f(x) = \begin{cases} x+2 & \text{if } x > 2 \\ -3 & \text{if } x = 2 \\ x^2 & \text{if } x = -1 < x < 2 \\ 5 & \text{if } x < -1 \end{cases}$



Domain $= \mathbb{R}$
Range $= \underbrace{\{ y \mid y = -3,\ y = \mathbb{R}^+ \setminus 4 \}}$

This reads as all y values such that y is -3 or any positive real number except 4.

9. $g(x) = \begin{cases} \frac{1}{x} & \text{if } x > 0 \\ -\frac{1}{2}x + 1 & \text{if } -4 < x \leq 0 \\ x^2 & \text{if } x \leq -4 \end{cases}$



DOMAIN $= \mathbb{R}$
Range $= \mathbb{R}^+$, the pos. real numbers.