Name: Solutions

- **Keep phones off and out sight.**
- **No calculators, notes, books, or other aids.**
- **Do not talk during the quiz.**
- **Show all work.**

1. Consider the function $f(x) = \dfrac{x}{x^2+4}$.

   (a) Determine the intervals on which $f(x)$ is increasing and decreasing.

$$f'(x) = \frac{1\cdot(x^2+4) - x\cdot 2x}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2} = -\frac{(x+2)(x-2)}{(x^2+4)^2}$$

numerator is 0 @ $x=-2$ & 2
denom. is never 0 $\because (x^2+4>0)$.

	$x<-2$	$-2<x<2$	$x>2$
$-\frac{1}{(x^2+4)^2}$	$-$	$-$	$-$
$(x+2)$	$-$	$+$	$+$
$(x-2)$	$-$	$-$	$+$
f'	$-$	$+$	$-$
f	↘	↗	↘

inc. on $(-2,2)$
dec. on $(-\infty,-2)$ & $(2,\infty)$

   (b) Find the x-coordinates of any local max(s) and min(s) of $f(x)$.

By 1^{st}. deriv. test, local min @ $x=-2$ (↘↗)
& local max @ $x=2$ (↗↘)

2. Consider the function $f(x) = x^3 - 3x^2 - 9x + 2$.

   (a) Find the x-coordinates of any local max(s) and min(s) of $f(x)$.

$$f'(x) = 3x^2 - 6x - 9$$
$$= 3(x^2 - 2x - 3)$$
$$= 3(x+1)(x-3)$$

$\Rightarrow$ crit. numbers -1 & 3
(f' never undefined)

	$(-\infty,-1)$	$(-1,3)$	$(3,\infty)$
$3(x+1)$	$-$	$+$	$+$
$x-3$	$-$	$-$	$+$
f'	$+$	$-$	$+$
f	$\nearrow$	$\searrow$	$\nearrow$

By 1st deriv. test,

$\boxed{\text{local max @ } x=-1}$ $(\nearrow\searrow)$

$\boxed{\text{local min @ } x=3}$ $(\searrow\nearrow)$

   (b) Find the intervals on which $f(x)$ is *concave up* and *concave down*.

$$f''(x) = 6x - 6$$
$$= 6(x-1)$$

zero at $x=1$, never undefined

	$(-\infty,1)$	$(1,\infty)$
$6(x-1)$	$-$	$+$
f	$\cap$	$\cup$

$\boxed{\text{conc. down on } (-\infty, 1)}$

$\boxed{\text{conc. up on } (1, \infty)}$

   (c) Find the x-coordinates of any inflection point(s) of $y = f(x)$.

$\boxed{x=1}$ (changes from conc. down to conc. up)