

Name: Solutions

- Keep phones off and out sight.
- No calculators, notes, books, or other aids.
- Do not talk during the quiz.
- Show all work.

1. Find the absolute maximum and absolute minimum of the function  $f(x) = 4x - x^2$  on the interval  $[0, 3]$ .

$$f'(x) = 4 - 2x$$

Crit. pts:

 $f'(x)$  undefined: nowhere.

$$f'(x) = 0$$

$$\Leftrightarrow 4 = 2x$$

$$\Leftrightarrow \underline{x=2}$$

Candidates: 0, 2, &amp; 3.

$$\text{bdy. } \begin{cases} f(0) = 0 \leftarrow \text{min} \\ f(3) = 12 - 9 = 3 \end{cases}$$

$$\text{crit. pt. } \begin{cases} f(2) = 8 - 4 = 4 \leftarrow \text{max} \end{cases}$$

on  $[0, 3]$ :  
 abn. max is 4 (@  $x=2$ )  
 abn. min is 0 (@  $x=0$ )

2. Find all critical numbers of the function  $f(x) = 3x^{2/3} - 4x$ . (Only the  $x$ -coordinate(s) are needed.)

$$f'(x) = 3 \cdot \frac{2}{3} x^{-1/3} - 4$$

$$= \frac{2}{x^{1/3}} - 4.$$

undefined at  $x=0$ 

$$f'(x) = 0$$

$$\Leftrightarrow \frac{2}{x^{1/3}} = 4$$

$$\Leftrightarrow \frac{1}{2} = x^{1/3}$$

$$\Leftrightarrow \frac{1}{8} = x$$

crit. numbers are  
 $x=0$  &  $x=1/8$

(problem 3 on reverse)

3. Find all critical numbers of the function  $f(x) = (x-2)^3(x-7)^2$ . (Only the  $x$ -coordinates are needed.)

*Hint: remember to look for common factors before expanding anything!*

$$\begin{aligned}f'(x) &= 3(x-2)^2(x-7)^2 + (x-2)^3 \cdot 2(x-7) \\&= (x-2)^2(x-7) \cdot [3(x-7) + 2(x-2)] \\&= (x-2)^2(x-7) \cdot [3x-21+2x-4] \\&= (x-2)^2(x-7) \cdot (5x-25) \\&= 5(x-2)^2(x-7)(x-5)\end{aligned}$$

undefined: nowhere.

$$f'(x)=0 \Leftrightarrow x-2=0, \quad x-7=0, \quad \text{or} \quad x-5=0.$$

$$\Leftrightarrow \boxed{x=2, 5, \text{ or } 7.}$$