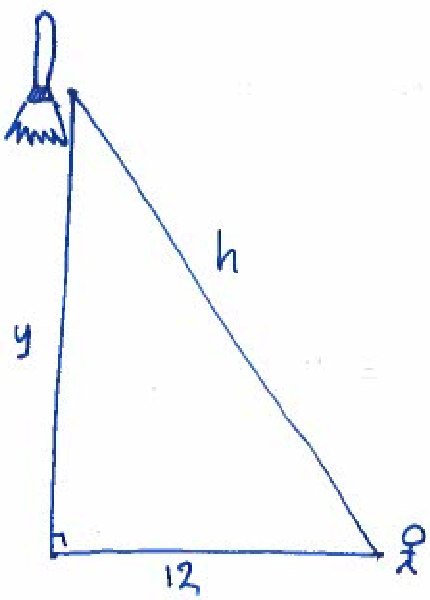
Name: Solutions

- **Keep phones off and out sight.**
- **No calculators, notes, books, or other aids.**
- **Do not talk during the quiz.**
- **Show all work.**

1. You are watching a rocket launch from a spot 12 kilometers away from the launchpad. The rocket flies straight up after launching. Shortly after launch, you use a radar instrument to determine that the rocket is 15 kilometers away from your location, and that the distance between you and the rocket is growing by $\frac{3}{10}$ of a kilometer per second. How quickly is the rocket rising at that instant?

   It may be useful to know that $\sqrt{15^2 - 12^2} = 9$.



variables:

y = rocket height

h = distance you → rocket

relation

$12^2 + y^2 = h^2$

Given: at key moment.

$h = 15$

$h' = 3/10$

want (at key moment)

$y' = ?$

which implies: $y = \sqrt{h^2 - 12^2} = \sqrt{15^2 - 12^2}$
$= 9.$

differentiate.

$$\frac{d}{dt}(12^2) + \frac{d}{dt}(y^2) = \frac{d}{dt}(h^2)$$

$$0 + 2y \cdot y' = 2h \cdot h'$$

solve at key moment.

$$2 \cdot 9 \cdot y' = 2 \cdot 15 \cdot \frac{3}{10}$$

$$\Rightarrow \quad y' = \frac{\cancel{2} \cdot 15 \cdot 3}{\cancel{2} \cdot 9 \cdot 10} = \frac{45}{90}$$

$$= 1/2.$$

$\boxed{\text{Rising at } 1/2 \text{ of a km. per second.}}$

(Question 2 is on the back)

2. A spherical balloon is being inflated by a pump. Its volume is increasing by ~~40~~ 400 cm^3 per second. How quickly is the radius of the balloon increasing (in cm per second) when the radius is 20cm?

   (The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.)

variables

$V =$ volume of balloon

$r =$ radius of balloon

equation

$$V = \frac{4}{3}\pi r^3$$

given info: at key moment,

$$V' = 40$$

$$r = 20$$

want (at key moment)

$$r' = ?$$

differentiate

$$V' = \underbrace{\frac{4}{3}\pi}_{\text{const. factor}} \frac{d}{dt}(r^3) = \frac{4}{3}\pi \cdot 3r^2 \cdot r'$$

$$= 4\pi r^2 \cdot r'$$

substitute & solve (at key moment)

$$400 = 4\pi \cdot 20^2 \cdot r'$$

$$= 4\pi \cdot 400\, r'$$

$$= 1600\pi \cdot r'$$

$$\Rightarrow \quad r' = \frac{400}{1600\pi} = \frac{1}{4\pi}$$

radius grows at $\frac{1}{4\pi}$ cm/sec.