Amherst College
Department of Mathematics and Statistics

Math 105 — Test #3 (slightly modified) — Fall 2015

Name:____

Read This First!

- This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, webpages, or other aids are permitted. Cell phones are to be out of sight.
- Please read each question carefully. Show **all** work clearly in the space provided.
- If you need addition space to do a problem, please use the back of the **previous** page.
- In order to receive full credit on a problem, solution methods must be complete, logical and understandable
- Answers must be clearly labeled.
- The exam consists of Questions 1–9, which total to 100 points.

Grading - For Instructor Use Only

Question:	1	2	3	4	5	6	7	Total
Points:	12	10	16	14	16	20	12	100
Score:								

1. [12 points] Let $f(x) = 3x^5 - 20x^3$.

   (a) Find the critical numbers of $f(x)$.

   (b) Test whether they are local maximums, local minimums, or neither.

2. [10 points] Consider a function $g(x)$ with the property that $g(2) = 3$, $g'(2) = 0$, and $g''(2)$ is some nonzero number. We are also given that $g(x)$ has a local maximum when $x = 2$. Determine whether $g''(2)$ positive is negative. Your solution should include an explanation (in words!) and a picture. Be sure to indicate how the words relate to the picture.

3. [16 points] The radius and height of a cylinder are changing with respect to time. The radius is increasing at a rate of 2 cm/sec, while the height is decreasing at a rate of 1 cm/sec. How fast is the volume of the cylinder changing at the instant of time when the radius is 10 cm and the height is 5 cm? (You may assume that the volume of a cylinder of radius r and height h is $V = \pi r^2 h$.)

4. [14 points] Find the absolute maximum and minimum values of the function

$$f(x) = x(x-4)^3$$

on the interval $[0, 3]$.

5. [16 points] Find where $g(x) = \dfrac{x}{(x+3)^2}$ is increasing and decreasing.

6. [20 points] The function $f(x) = \dfrac{1+x^2}{x^2-4}$ has first and second derivatives given by:

$$f'(x) = \frac{-10x}{(x^2-4)^2}, \qquad f''(x) = \frac{10(3x^2+4)}{(x^2-4)^3}.$$

(a) Use this information to determine where $y = f(x)$ is increasing or decreasing, and find any local max(s) or local min(s).

(b) Use this information to determine where $y = f(x)$ is concave up or concave down.

(c) Compute $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. Use these values to identify any *horizontal asymptotes* of the graph $y = f(x)$.

7. [12 points] The previous problem stated that $f(x) = \dfrac{1+x^2}{x^2-4}$ has derivatives

$$f'(x) = \frac{-10x}{(x^2-4)^2}, \qquad f''(x) = \frac{10(3x^2+4)}{(x^2-4)^3}.$$

Verify that these formulas are correct by computing $f'(x)$ and $f''(x)$ using the usual rules of differentiation.