Math 105   Practice Exam #3   Fall 2013 Compact Version

• This is a closed-book examination. No books, notes, calculators, cell phones, communication devices of any sort, or webpages, or other aids are permitted.

• Please *show* all of your work and *justify* all of your answers.

Critical Numbers

1. Find critical numbers for the function $f(x) = x^{\frac{1}{3}}(8 - x)$.

2. Find critical numbers for the function $f(x) = \dfrac{2x^3 + x^2 - 1}{x^3}$.

Absolute Extreme Values

3. Find the absolute maximum and absolute minimum values of

$$F(x) = x\sqrt{4 - x^2} \quad \text{on} \ \ [-1, 2].$$

4. Find the absolute maximum and absolute minimum values of

$$G(x) = x^3 + 6x^2 - 1 \quad \text{on} \quad [-1, 1].$$

Related Rates

5. Suppose a 20 foot ladder is sliding down a vertical wall. The base of the ladder is sliding on the level ground, away from the wall, at 2 feet per second. At what rate is the top of the ladder sliding down after 5 seconds has passed?

6. A conical paper cup of water is 4 inches across the top and 5 inches deep. It has a hole in the bottom point and is leaking water at 2 inches per second. At what rate is the height of the water decreasing when the water height is 1 inch?

Limits at Infinity

7. Compute each of the following limits at infinity. Please show your work.

(a) $\displaystyle\lim_{x\to\infty} \frac{x^3 - 5x^2 - 90}{-9x^3 - 6x^2 + 4}$

(b) $\displaystyle\lim_{x\to\infty} \frac{x^2 - x + 1}{2x^5 + 7x^2 + 3}$

(c) $\displaystyle\lim_{x\to\infty} \frac{x^{99} + 99}{100x^{98} + x + 97}$

Curve Sketching For each of the following functions, discuss domain, vertical and horizontal asymptotes, intervals of increase or decrease, local extreme value(s), concavity, and inflection point(s). Then use this information to present a detailed and labelled sketch of the curve.

8. $f(x) = x^4 - 6x^2$

9. $f(x) = \dfrac{3x^2}{1 - x^2}.$ Take my word for it that (you do NOT have to compute these)

$$f'(x) = \frac{6x}{(1 - x^2)^2} \text{ and } f''(x) = \frac{6(1 + 3x^2)}{(1 - x^2)^3}.$$