



1. [15 points] Compute the following derivatives.

(a)  $\frac{d}{dx} \left( \frac{x^2 + \pi^2}{x^3 + \sqrt{7}^3} \right)$ . Do *not* simplify your answer.

(b) Let  $f(u) = \frac{h(u)}{u^2 + 1}$ , where  $h(2) = -1$  and  $h'(2) = 3$ . Compute  $f'(2)$ .

(c)  $\left( (x^2 + 1)^3 (1 - 3x)^2 \right)'$ . Do *not* simplify your answer.

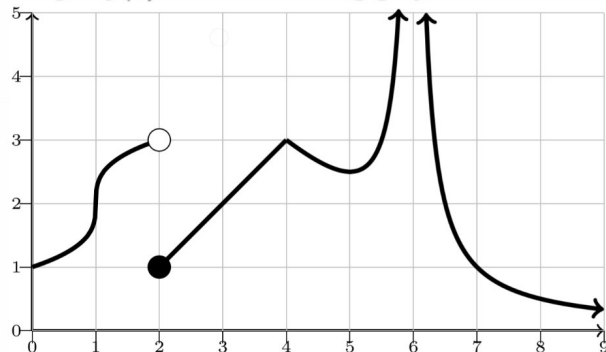
2. [14 points]

(a) State the limit definition of the derivative of a function  $f(x)$ .

(b) Compute  $\frac{d}{dx}\left(\frac{1}{x^2+1}\right)$  using the limit definition of derivative.

3. [10 points] Compute the second derivative of  $f(x) = \frac{x^2}{x+3}$  and simplify your answer.

4. [15 points] Suppose that  $y = f(x)$  has the following graph:



(a) For which numbers  $a$  does  $f(x)$  fail to be continuous at  $a$ ? Give reasons using the definition of continuity?

(b) For which numbers  $a$  does  $f(x)$  fail to be differentiable at  $a$ ? Give reasons.

(c) Find all  $x$ 's for which  $f'(x) > 0$ .

5. [16 points] We are adding trash to a brand new landfill. Assume that the amount of trash in the landfill at time  $t$  (= months since the landfill opened) is given by the formula

$$W(t) = 100t + 10t^2 \text{ tons of trash.}$$

- (a) How much trash was added to the landfill during the first six months of its operation?

- (b) Compute the rate of adding trash during this six month time period.

- (c) What was the rate of adding trash exactly six months after the landfill opened?

- (d) When you compare the answers to parts (b) and (c), what conclusion do you draw?

6. [10 points] Find the equation of the line tangent to the curve  $y = \frac{x^2 + \sqrt{x} + 1}{2 - x}$  at the point where the  $x$ -coordinate is equal to 1.

7. [15 points] Let

$$f(x) = \frac{x^2}{\sqrt{x^2 - 1}}.$$

Note that  $f(x)$  is defined when  $x^2 > 1$ , which holds when either  $x > 1$  or  $x < -1$ .

- (a) Compute  $f'(x)$  and simplify your answer as much as possible. Your final answer should be  $f'(x) = \frac{x(x^2 - 2)}{(x^2 - 1)^{3/2}}$ . To get full credit, I need to see every step of the simplification.

- (b) Find all points on the curve where the tangent line is horizontal.



8. [5 points] The production  $q$  of a company depends on both the capital investment  $K$  (in dollars) and the size of the labor force  $L$  (the number of workers). In economics, one frequently used formula for  $q$  in terms of  $K$  and  $L$  is the *Cobb-Douglas production function*

$$q = \sqrt{KL}.$$

Assuming the capital investment remains constant, compute the rate of change of production as the number of workers increases.

