1. [21 points] Evaluate each limit.

   (a) $\displaystyle\lim_{x\to 7} \frac{x^2-8x+7}{x^2-6x-7}$

   (b) $\displaystyle\lim_{x\to 1} \frac{\sqrt{4-3x}-1}{x^2-1}$

   (c) $\displaystyle\lim_{x\to 4^-} \frac{|x-4|}{x^2-3x-4}$

   (d) $\displaystyle\lim_{x\to -\infty} \frac{3-4x-4x^2}{9+2x^2}$

   (e) $\displaystyle\lim_{x\to \infty} \frac{x^3-10x^2}{5x^2+7}$

   (f) $\displaystyle\lim_{x\to 8} \frac{\frac{x}{x+4}-\frac{x-4}{x-2}}{x^2-10x+16}$

   (g) $\displaystyle\lim_{x\to 2^+} \frac{x^2-x-6}{x^2+x-6}$

2. [15 points] Evaluate the **derivative** of each function. You do not need to simplify your answer.

   (a) $f(x) = (3x-7)\left(x^{1/3}+\dfrac{1}{x^4}\right)$

   (b) $g(x) = \left(2x^3+5x^4\right)^{1/3}$

   (c) $h(x) = (x^2+7)\sqrt{5x+3}$

   (d) $f(x) = \dfrac{\sqrt{2x+3}}{x^2+1}$

   (e) $g(x) = (2x-1)^3(5x+3)^5$

3. [9 points] Let $f(x) = \dfrac{2x}{3x+1}$. Compute $f'(x)$ using the **limit definition** of the derivative. You may use the quotient rule to check your answer, but for full points all steps of the limit calculation must be shown.

4. Consider the curve defined by the equation

$$y^2 = x^3 - x + 1.$$

   (a) [4 points] Determine $\dfrac{dy}{dx}$ using **implicit differentiation.** Your answer will be in terms of both x and y.

   (b) [4 points] Find the equation of the tangent line at the point $(3, 5)$.

5. [9 points] Find the absolute maximum and absolute minimum values of $f(x) = x^2(x-5)^3$ on the interval $[0, 6]$.

6. [12 points] Consider the following function.

$$f(x) = \frac{2-x}{x^2}$$

   Sketch the graph $y = f(x)$. Clearly label the following features on your graph: asymptotes (horizontal or vertical), intervals where it is increasing/decreasing, intervals where it is concave up/down, local max(s) and min(s), and inflection point(s).

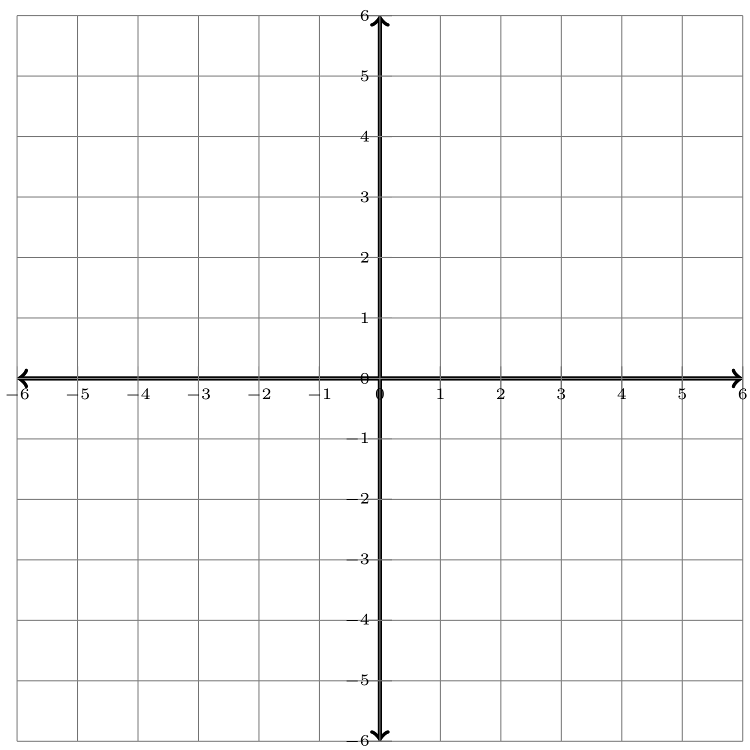
7. [12 points] A 15 foot ladder is leaning against a wall, and sliding down the side. At this moment, the bottom of the ladder is 9 feet away from the wall, and is sliding away at 2 feet per second. Determine how quickly the top of the ladder is sliding down the wall at this moment (in feet per second).

8. [12 points] A small rectangular box with a square base **with no lid** is to be constructed out of 12 square inches of cardboard. Determine what dimensions the box should have in order for its volume to be as large as possible.

9. Consider the following piecewise function.

$$f(x) = \begin{cases} 2 & x < 0 \\ \frac{1}{x-2} & 0 \le x < 2 \\ 2\sqrt{x-2} & x \ge 2 \end{cases}$$

   (a) [6 points] Sketch the graph $y = f(x)$ on the axes below (for this sketch, you don't need to take any derivatives or apply the techniques from Chapter 3; instead think about how each piece of the graph is obtained from a graph you already know about).



   (b) [6 points] Evaluate each of the following quantities (no explanation or scratchwork is required).

- $\lim_{x\to 0^-} f(x)$
- $\lim_{x\to 0^+} f(x)$
- $f(0)$
- $\lim_{x\to 2^-} f(x)$
- $\lim_{x\to 2^+} f(x)$
- $f(2)$

(c) [3 points] Determine all points where $f(x)$ is discontinuous.

10. [12 points] Consider the function

$$f(x) = \frac{x}{4 + x^2}.$$

(a) Compute $f'(x)$, and simplify.

(b) Determine all critical numbers of $f(x)$.

(c) Determine the intervals on which $f(x)$ is increasing and decreasing.

(d) Classify each critical point as a local minimum, a local maximum, or neither.